CIV102 — Units & Formula Cheat Sheet

Use N, mm, MPa consistently

Part 1 — Units & Core Formulas (CIV102)

CIV102 Standard Unit System

- Force: N or kN
- Length: mm (preferred)
- Area: mm^2
- Second moment of area: mm^4
- Stress: MPa
- Moment: N·mm or kN·m
- $\boxed{1\ \text{MPa} = 1\ \text{N/mm}^2}$

Key Conversions

- $1\ \text{m} = 1000\ \text{mm}$
- $1\ \text{m}^2 = 10^6\ \text{mm}^2$
- $1\ \text{m}^3 = 10^9\ \text{mm}^3$
- $1\ \text{kN} = 10^3\ \text{N}$
- $1\ \text{kN·m} = 10^6\ \text{N·mm}$
- $1\ \text{kN/m} = 1\ \text{N/mm}$
- $1\ \text{MPa} = 10^6\ \text{Pa}$
- $1\ \text{GPa} = 10^3\ \text{MPa}$

Equations of Equilibrium (2D)

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M = 0$$

For static structures (no acceleration).

Stress (Axial)

$$\sigma = \frac{F}{A}$$

- F in N, A in mm^2
- σ in MPa

Strain

$$\varepsilon = \frac{\Delta L}{L}$$

- Unitless (mm/mm or %)

Hooke's Law

$$\sigma = E\varepsilon$$

- E in MPa or GPa

Axial Deformation

$$\delta = \frac{FL}{AE}$$

- F in N, L in mm
- A in mm^2, E in MPa
- δ in mm

Moment (Definition)

$$M = Fd$$

- M in N·mm or kN·m

Bending Stress (Navier's Equation)

$$\sigma = \frac{My}{I}$$

- M in N·mm
- y in mm
- I in mm^4
- σ in MPa

Beam Curvature

$$\kappa = \frac{M}{EI}$$

- E in MPa, I in mm^4
- κ in mm^{-1} (multiply by 1000 for m^{-1})

Shear Stress in Beams

$$\tau = \frac{VQ}{It}$$

- V in N
- Q in mm^3
- I in mm^4, t in mm
- τ in MPa

Gravity

$$g = 9.81\ \text{m/s}^2 = 9810\ \text{mm/s}^2$$

Fast Unit Checks

$$\frac{N}{\text{mm}^2} = \text{MPa} \qquad \frac{N \cdot \text{mm}}{\text{mm}^3} = \text{MPa} \qquad \frac{M}{EI} = \text{curvature}$$

Part 2 — Truss Analysis & Design (CIV102)

Loads & Tributary Areas

Total gravity load on deck:

$$w_{\text{total}} = w_{\text{deck}} + w_{\text{struct}} + w_{\text{live}}$$

Typical values:

$$w_{\text{live}} = 5.0\,\text{kPa}, \quad w_{\text{deck}} \approx 1.0\,\text{kPa}, \quad w_{\text{struct}} \approx 0.5\text{–}1.0\,\text{kPa}$$

Joint load from tributary area:

$$P_i = w_{\text{total}} A_{\text{trib}}$$

Equilibrium (Trusses)

Global equilibrium:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$

Member force components:

$$F_x = F\cos\theta = \frac{a}{c}F, \quad F_y = F\sin\theta = \frac{b}{c}F$$

Sign convention:

$$\text{Tension} = +, \quad \text{Compression} = -$$

Method of Joints

At each joint:

$$\sum F_x = 0, \quad \sum F_y = 0$$

Max unknowns per joint:

$$\boxed{2}$$

Method of Sections

Cut through max 3 unknown members:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$

Moment trick:

Take moments about point where 2 unknowns pass

Euler Buckling

Crushing (short members):

$$P_{\text{crush}} = \sigma_{\text{crush}} A$$

Euler buckling load:

$$P_e = \frac{\pi^2 EI}{L^2}$$

Buckling stress:

$$\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E}{(L/r)^2}$$

Radius of gyration:

$$r = \sqrt{\frac{I}{A}}$$

Slenderness ratio:

$$\lambda = \frac{L}{r}$$

Design of Members

Stress in axial member:

$$\sigma = \frac{F}{A}$$

Tension members (yielding):

$$A \geq \frac{FOS_{\text{yield}} F}{\sigma_y} = \frac{2.0\,F}{\sigma_y}$$

Compression members (buckling):

$$I \geq \frac{FOS_{\text{buckling}} F L^2}{\pi^2 E} = \frac{3.0\,F L^2}{\pi^2 E}$$

Material properties (steel):

$$\sigma_y = 350 \text{ MPa}, \quad E = 200{,}000 \text{ MPa}$$

Slenderness limit:

$$\frac{L}{r} \leq 200$$

Wind Loads

Wind force:

$$F_{\text{wind}} = \frac{1}{2}\rho v^2 c_D A$$

Design wind pressure:

$$w_{\text{wind}} = 2.0 \text{ kPa}$$

Wind joint load:

$$P_{\text{wind}} = w_{\text{wind}} A_{\text{frontal}}$$

Frontal area (members):

$$A_{\text{frontal}} \approx \sum b_i l_i$$

Frontal area (handrail):

$$A_{\text{frontal}} \approx h_{\text{rail}} s_{\text{bot}}$$

Bracing & Stability

Allowable joint misalignment:

$$\Delta = 0.01L$$

Required restraint force:

$$R = 0.02P$$

Braces must be designed for:

Tension AND Compression

Load cases:

Wind & instability checked separately

Part 3 — Flexural Members (Beams)

Beam Theory Assumptions

Plane sections remain plane.
Linear elastic material:

$$\sigma = E\varepsilon$$

Pure bending (no axial force):

$$\int y\, dA = 0$$

Curvature & Navier's Equation

Strain distribution:

$$\varepsilon(y) = \phi y$$

Stress distribution:

$$\sigma(y) = E\phi y$$

Moment–curvature relation:

$$M = EI\phi$$

Navier's Equation:

$$\boxed{\sigma(y) = \frac{My}{I}}$$

Maximum stresses:

$$\sigma_{\max} = \frac{My_{\max}}{I} = \frac{M}{S}$$

Section modulus:

$$S = \frac{I}{y_{\max}}$$

Radius of gyration:

$$r = \sqrt{\frac{I}{A}}$$

Centroid of Composite Sections

Location of centroidal axis:

$$\boxed{\bar{y} = \frac{\sum y_{i,b} A_i}{\sum A_i}}$$

Total area:

$$A = \sum A_i$$

Second Moment of Area

Definition:

$$I = \int y^2\, dA$$

Composite sections:

$$I = \sum I_i$$

Parallel Axis Theorem:

$$\boxed{I_i = I_{o,i} + A_i d_i^2}$$

Total:

$$I = \sum (I_{o,i} + A_i d_i^2)$$

Common shapes:

$$I_{\text{rect}} = \frac{bh^3}{12} \qquad I_{\text{circle}} = \frac{\pi d^4}{64}$$

Symmetric sections:

$$I = I_{\text{out}} - I_{\text{in}}$$

Stress Resultants

Axial force:

$$N$$

Shear force:

$$V$$

Bending moment:

$$M$$

Equilibrium at section:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$

Shear Force & Bending Moment Diagrams

Load–shear relationship:

$$\boxed{w(x) = \frac{dV}{dx}}$$

Shear–moment relationship:

$$\boxed{V(x) = \frac{dM}{dx}}$$

Change in shear:

$$\Delta V = \int_A^B w(x)\, dx$$

Change in moment:

$$\Delta M = \int_A^B V(x)\, dx$$

Key heuristics:

- Point load $\to$ jump in V
- Distributed load $\to$ slope in V
- Zero shear $\to$ maximum M
- Moment at pin/roller $= 0$
- Moment at internal hinge $= 0$

Sign convention:

- Positive shear $\to$ upward on left face
- Positive moment $\to$ bottom fibre in tension

Deflection & Moment Area Theorems

Curvature:

$$\phi(x) = \frac{M(x)}{EI}$$

Moment Area Theorem 1 (Slope):

$$\boxed{\Delta\theta_{AB} = \int_A^B \phi(x)\, dx}$$

Moment Area Theorem 2 (Deflection):

$$\boxed{\delta_{DT} = \int_D^T x\phi(x)\, dx}$$

Interpretation:

- MAT1 $\to$ area under curvature diagram
- MAT2 $\to$ first moment of curvature area

Common Deflection Results

Cantilever, tip load P:

$$\theta_{\text{tip}} = \frac{PL^2}{2EI} \qquad \Delta_{\text{tip}} = \frac{PL^3}{3EI}$$

Simply supported, midspan load P:

$$\Delta_{\max} = \frac{PL^3}{48EI}$$

Shear Stress in Beams

Average shear stress:

$$\tau_{\text{avg}} = \frac{V}{A}$$

Jourawski's Equation:

$$\boxed{\tau = \frac{VQ}{Ib}}$$

Where:

$$Q = \int y\, dA$$

Rectangular section:

$$\tau_{\max} = \frac{3V}{2A}$$

Shear stress:

- Zero at top and bottom
- Maximum at neutral axis

Part 4 — Concrete Structures (CIV102)

Axial Load + Bending (Concrete)

Axial stress:

$$f_N = -\frac{N}{A}$$

Bending stress (Navier):

$$f_M = \frac{My}{I}$$

Combined stress:

$$f = -\frac{N}{A} \pm \frac{My}{I}$$

For self-weight:

$$N = V_0\gamma \qquad f = h\gamma$$

Material Properties

Concrete tensile strength:

$$f'_t = 0.33\sqrt{f'_c}$$

Concrete modulus:

$$E_c = 4730\sqrt{f'_c} \quad \text{(MPa)}$$

Steel modulus:

$$E_s = 200{,}000 \text{ MPa}$$

Steel yield stress (CIV102):

$$f_y = 400 \text{ MPa}$$

Modular ratio:

$$n = \frac{E_s}{E_c}$$

Cracked Reinforced Concrete (Flexure)

Curvature:

$$\phi = \frac{\varepsilon_{c,\text{top}}}{kd}$$

Concrete strain:

$$\varepsilon_{c,\text{top}} = \phi kd$$

Steel strain:

$$\varepsilon_s = \phi d(1-k)$$

Concrete compression force:

$$C_c = \frac{1}{2}bkdE_c\varepsilon_{c,\text{top}}$$

Steel tension force:

$$T_s = E_s\varepsilon_s A_s$$

Equilibrium:

$$C_c = T_s$$

Reinforcement ratio:

$$\rho = \frac{A_s}{bd}$$

Neutral axis factor:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

Flexural lever arm:

$$jd = d\left(1 - \frac{k}{3}\right) \qquad j = 1 - \frac{k}{3}$$

Moment resistance:

$$M = A_s f_s jd$$

Steel stress:

$$f_s = \frac{M}{A_s jd}$$

Concrete stress:

$$f_c = \frac{k}{1-k}\frac{M}{nA_s jd}$$

Yield moment:

$$M_y = A_s f_y jd$$

Design Limits (CIV102)

Steel stress limit:

$$f_s \le 0.6 f_y$$

Concrete stress limit:

$$f_c \le 0.5 f'_c$$

Minimum steel (estimate):

$$A_{s,\min} = \frac{M}{0.6 f_y jd}$$

Shear in Reinforced Concrete

Shear stress:

$$v = \frac{V}{b_w jd}$$

Maximum shear stress:

$$v_{\max} = 0.25 f'_c$$

Maximum shear force:

$$V_{\max} = 0.25 f'_c b_w jd$$

Design shear resistance:

$$V_r = 0.5V_c + 0.6V_s \le 0.5V_{\max}$$

Concrete Shear (No Stirrups)

$$V_c = \frac{230\sqrt{f'_c}}{1000 + 0.9d}\, b_w jd$$

Shear Reinforcement

Steel shear resistance:

$$V_s = \frac{A_v f_y jd}{s}\cot 35^\circ$$

Minimum shear reinforcement:

$$\frac{A_v f_y}{b_w s} \ge 0.06\sqrt{f'_c}$$

If minimum met:

$$V_c = 0.18\sqrt{f'_c}\, b_w jd$$

Stirrup Spacing

Required spacing:

$$s = \frac{0.6A_v f_y jd \cot 35^\circ}{V - 0.5(0.18\sqrt{f'_c}\, b_w jd)}$$

Prestressed Concrete

Concentric tendon stresses:

$$f_{c,\text{top}} = -\frac{P}{A} - \frac{My}{I}$$

$$f_{c,\text{bot}} = -\frac{P}{A} + \frac{My}{I}$$

Eccentric tendon stresses:

$$f_{c,\text{top}} = -\frac{P}{A} + \frac{Pey_{\text{top}}}{I} - \frac{My_{\text{top}}}{I}$$

$$f_{c,\text{bot}} = -\frac{P}{A} - \frac{Pey_{\text{bot}}}{I} + \frac{My_{\text{bot}}}{I}$$

Cracking check:

$$|f_c| \le f'_t$$