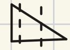
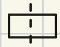
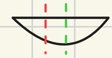
Beams

$\bar{x}$ = Distance from centroid to point of interest

Triangle: $\frac{2}{3} \cdot L$ from 0 end or $\frac{1}{3}$ max 

Rectangle: $\frac{1}{2} \cdot L$ 

Parabola (UDL): if max at L/2, centroid is at L/2, if not it's $\frac{3}{8} L$ from the zero-moment end where the diagram starts/$\frac{5}{8}L$ from max 

trapezoid $= \dfrac{b(2M_2 + M_1)}{3(M_1 + M_2)}$

M_1 = rectangle

M_2 = triangle

Beam Deflection/finding θ

Boils down to three cases

For all of them, start with:

- Find reaction forces
- Draw the BMD. Don't bother drawing curvature, when you integrate take $\frac{1}{EI}$ out
- Compute the area $\Rightarrow$ if no UDL, split up into simple shapes, if UDL use similar triangles/piecewise function
  Examples with UDL find area by defining a function and integrating $\Rightarrow$ you will not be asked this on an exam

Case 1 $\Rightarrow$ Known Horizontal tangent due to condition

Fixed end (cantilever), slope/angle at this point is 0

Since a known point has an angle of 0, let's call this point A we can compute the angle at point B using MAT 1

$$\text{MAT 1} \Rightarrow \underbrace{\theta_{AB}}_{\text{change}} = \theta_B - \theta_A = \theta_B - \underbrace{0}_{\text{fixed end}} = \int_A^B \frac{M}{EI}dx = \boxed{\frac{1}{EI} \cdot \int_A^B M(x)\,dx} = \theta_B$$

Deflection:

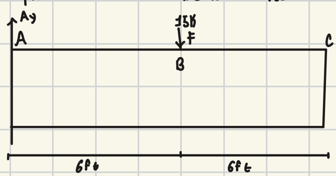
Using the same logic, point A is a fixed end, meaning it has no deflection. We can compute the deflection at point B using MAT 2

$$\text{MAT 2} \Rightarrow \underbrace{\Delta AB}_{\text{change}} = \Delta B - \Delta A = \Delta B - 0 = \int_A^B \frac{M}{EI} \cdot \bar{x}\,dx = \boxed{\frac{1}{EI} \cdot \int_A^B M(x) \cdot \bar{x}\,dx}$$

$\Rightarrow$ Deflection, $\bar{x}$ is the distance from centroid (see note top right to find centroids

$\bar{x} = |\text{centroid} - x_B|$

Example : 8-10 Beam Deflection Pset



$E = 29 \cdot 10^3$ ksi $\rightarrow 29 \cdot 10^6$ psi

$I = 500 in^4$

6ft $\Rightarrow$ 72 inches

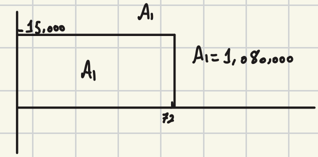
One reaction force $\Sigma F_y = 0$

$A_y - F = 0$

$A_y = F$

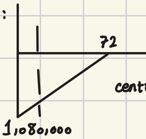
$A_y = 15K$

SFD:



$A_1 = 1{,}080{,}000$

BMD:



centroid $= \frac{1}{3}(72) = 24$

$$\text{Slope at } B = \int_A^B \frac{M}{EI}dx = \frac{1}{EI}\int_A^B M(x)dx$$

0 slope

$$\theta_B = \left(\frac{1}{(29 \cdot 10^6)(500)}\right)\left(\frac{1}{2} \cdot 72 \cdot 1{,}080{,}000\right)$$

$$= \frac{38{,}880{,}000}{(29 \cdot 10^6)(500)} = \boxed{0.00268 \text{ rad}}$$

Reminder for centroid: POINT OF REFERENCE

if you are measuring from A, that's the max, use $\frac{1}{3}$, if you did B, that's the zero end. Use $\frac{2}{3}$.

Max Deflection: if symmetric, always in the middle (or where BMD is max)

for fixed ends, it's at the free ends

otherwise, it's when $\theta = 0$

here, it's at point C

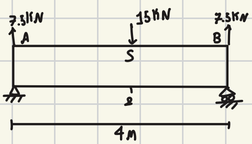
same logic as θ_B

$$\delta_C = \frac{1}{EI}\int_A^C M(x) \cdot \bar{x}\,dx = \frac{1}{(29 \cdot 10^6)(500)} \cdot (38{,}880{,}000)(120) = \boxed{0.322 \text{ in}}$$

$\bar{x} = 144 - 24 = 120$

Case 2 $\Rightarrow$ Known Horizontal Tangent due to symmetry

We can't identify θ at the supports, but if the beam is symmetrical, then θ at the symmetry point $S = 0$



$E = 2$ MPa

$I = 1 \times 10^6 mm^4$

$\Sigma M_A = 0$

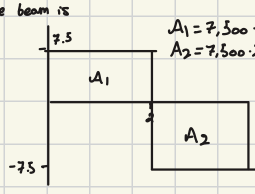
$-15{,}000 \cdot 2 + B_y \cdot 4 = 0$

$B_y \cdot 4 = 30{,}000$

$B_y = 7500$ N

$A_y = 7500$ N

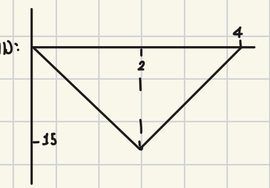
$\theta_S = 0$



$A_1 = 7{,}500 \cdot 2 = 15{,}000$

$A_2 = 7{,}500 \cdot 2 = 15{,}000$

BMD:



$\theta_{AS} = \theta_S - \theta_A = -\theta_A = \int_A^S \frac{M}{EI}dx = \frac{1}{EI}\int_0^2 M(x)dx \Rightarrow \theta_A = \frac{-1}{(200{,}000)(1\cdot 10^6)}\cdot\left(\frac{1}{2}\cdot 2\cdot 15{,}000\right) = \boxed{-7.5\times 10^{-8}\text{ rad}}$ θ_A, by symmetry $\Rightarrow \boxed{\theta_B = 7.5\times 10^{-8}\text{ rad}}$

Deflection: $\delta_{AS} = \int_A^S \frac{M}{EI}\cdot\bar{x}\,dx$ we are only considering the first triangle, so $\bar{x}$ is $\frac{1}{3}(2) = \frac{2}{3}$m

we already computed $\int_A^S \frac{M}{EI}dx = -7.5\times 10^{-8} \Rightarrow \delta_{AS} = -7.5\times 10^{-8}\cdot \underbrace{\frac{2}{3}}_{\bar{x}} = -5\times 10^{-8}$ mm

Case 3 ⇒ No known horizontal tangent

Beam is not fixed and is asymmetric

Let A = first support, C = second support

$\delta_A = 0, \delta_C = 0$

- Use MAT 1, determine θ_{AC}

Re-arranging MAT 2 we get that

$\delta_{AC} = \delta_C - \delta_A = \theta_A L - \int_A^C \frac{M(x)}{EI}\bar{x}$

Double integral derivation, just know it's always true

since $\delta_{AC} = 0(0-0) \Rightarrow 0 = \theta_A \cdot L - \delta_{AC}$

$\boxed{\theta_A = \frac{\delta_{AC}}{L}}$

Since θ_A is defined, pick any point B any apply MAT2

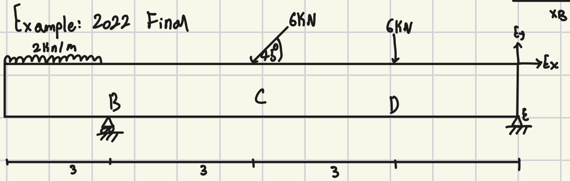
$\delta_{AB} = \delta_B - \delta_A = \theta_A \cdot x_B - \int_A^B \frac{M(x)}{EI}\bar{x}\,dx$

since A is a support

$\boxed{\delta_B = \theta_A \cdot x_B - \int_A^B \frac{M(x)}{EI}\bar{x}\,dx}$

x_B = distance from A to B

Example: 2022 Final



Goal: Deflection at point B $6\cos 45 = 4.24$

Case 3

$\Sigma M_B = 0$

$0 = (-3\cdot -2\cdot 1.5) - (4.24\cdot 3) - (6\cdot 6) + E_y\cdot 9$

$E_y = 4.41\uparrow$

$\Sigma F_y = 0$

$0 = 4.41 + B_y - 6 - 4.24 - 6$

$B_y = 11.83$

$\Sigma F_x = 0$

$0 = 4.24 - E_x$

$E_x = 4.24$

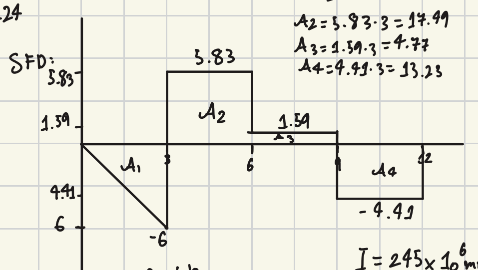
$A_1 = \frac{1}{2}(6)(3) = 9$

$A_2 = 5.83\cdot 3 = 17.49$

$A_3 = 1.59\cdot 3 = 4.77$

$A_4 = 4.41\cdot 3 = 13.23$

SFD:

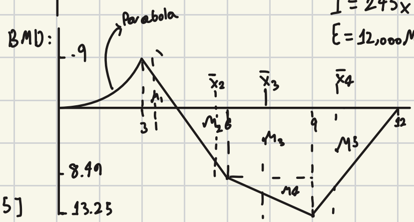


$I = 245\times 10^6\text{ mm}^4$

$E = 12{,}000$ MPa

$\theta_B = \frac{\delta_{BE}}{L}$

BMD:



$\delta_{BE} = \frac{1}{EI}\int_B^E M(x)\,\bar{x}\,dx$

$\delta_{BE} = \frac{1}{EI}\left[-13.5\cdot 2 + 12.75\cdot 2 + 32.625\cdot 2.36 + 19.875\cdot 5\right]$

$\delta_{BE} = \frac{1}{EI}\cdot(174.87e^{12}) = 59.64$

I messed up xbar somewhere this should be 39.47

afterwards divide by L, find θ_B

Find θ_B, then use $\delta_D = \theta_B\cdot x_D \cdot \int_B^B \frac{M(x)}{EI}\bar{x}$

I think I also had a unit conversion error

$A'_1 = M_1 = \frac{1}{2}(9)(3) = 13.5$

$A'_2 = M_2 = \frac{1}{2}(3)(8.49) = 12.735$

$A'_3 = M_3 + M_4 = (3)(8.49) + \frac{1}{2}(3)(4.44) = 32.625$

$A'_4 = M_5 = \frac{1}{2}(13.25)(3) = 19.875$

$\bar{x}_1 = \frac{1}{3}(3) = 1 \Rightarrow 3 - 1 = 2$

$\bar{x}_2 = 3 + \frac{2}{3}(3) = 5 \Rightarrow 5 - 3 = 2$

$\bar{x}_3 = 7.36 \Rightarrow 7.36 - 3 = 4.36$

$\bar{x}_4 = 9 + (\frac{1}{3}) = 10 \Rightarrow 10 - 5 = 5$

$x_4 = \frac{\ }{3} = 10$

$\tau = \frac{VQ}{IB} \Rightarrow$ Q = First moment of area ($\Sigma A_i d_i$)
V = Shear Force
I = moment of inertia
B = depth of intrest

max is about the Centroid, use abs of the max SFD)

if $\Delta L < \frac{L}{100} \Rightarrow$ Acceptable

ν = poisson's ratio

- Plate buckling equations are given below:

Table 30.2 – Summary of plate buckling failure modes

No.	Failure Mode	Failure Condition	Relevant Design Equation
5	Buckling of the compressive flange between the webs	$\sigma = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	
6	Buckling of the tips of the compressive flange	$\sigma = \frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
7	Buckling of the webs due to the flexural stresses	$\sigma = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	
8	Shear buckling of the webs	$\tau = \frac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right)$	$\tau = \frac{VQ}{Ib}$

For $\frac{My}{I}$, consider Max Moments (+ve, -ve)
y_{top} = distance from top to centroid
y_{bot} = distance from bottom to centroid

Combined flexural + axial:
$\left(\frac{My}{I} + \frac{P}{A}\right)$

$\frac{My}{I} \Rightarrow$ +ve for tension, -ve for comp } use values computed before

$\frac{P}{A} \Rightarrow$ Same sign for both, P is axial force
P contributes to our stress based on sign

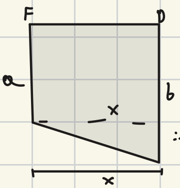
Maximum Displacement
Let the location of the max displacement be F
at F, $\theta_F = 0$
$\therefore \theta_C - \theta_F$ = (Area between F, C)

support, find it's angle

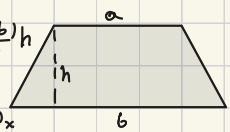
$\theta_C - 0 = A_1 + A_{F-D}$

should have a general idea of where it will be, pick the closest point after call it D

if we had something like this



$A = \frac{(a+b)}{2} h$



$\therefore A_{F-D} = \frac{(a+b)}{2} x$

$\therefore \theta_C = A_1 + A_{F-D}$ isolate for x from here

Vibrations:

$$DAF = \frac{1}{\sqrt{\left(1-\left(\frac{f}{f_n}\right)^2\right)^2 + \left(\frac{2\beta f}{f_n}\right)^2}}$$

β : Damping ratio (0.02)
f = driving frequency
f_n = natural frequency

$W_{tot} = W_{stationary} + D.A.F \cdot w_0$
$w_0 \pm$ uncertainty

$f_n = \frac{17.76}{\sqrt{\Delta_{new}}}$ if point load

if UDL: $f_n = \frac{15.36}{\sqrt{\Delta_{new}}}$

ΔC = deflection at point C

$\Delta_{new} = Z \cdot \Delta_{old}$
$\Delta_{new} \Rightarrow$ new dead load
$\Delta_{old} \Rightarrow$ original force
Z = ratio $\left(\frac{w_{new}}{w_{old}}\right)$

$\Delta_{max} = \Delta_{old} \cdot Z'$
$Z' = \frac{W_{tot}}{W_{old}}$
old force

Trusses:
$\Delta C = \frac{\Sigma F^* \cdot F \cdot L}{EA}$

Axial Tension:
$A \geq \frac{2 \cdot F}{\sigma_y}$

Axial comp
$A \geq \frac{2 \cdot F}{\sigma_y}$

Buckling:
$I \geq \frac{8 \cdot F L^2}{\pi^2 \cdot E}$